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FIRST AND LAST OCCURRENCES OF LOW TEMPERATURES DURING THE COLD SEASON

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ABSTRACT

Current data analysis methods leading to a climatological probability forecast of the first fall and last spring occurrences of preselected low temperatures do not work well in Florida, the Gulf Coast States, and the Southeastern Atlantic States where these seasons are not well defined. A proposal is made to recognize only a "cold season" beginning July 1 and ending June 30 and to construct data series of the first and last cold season events of low temperatures of interest to assign climatological probabilities to the dates of their occurrence. This proposal eliminates the analytic difficulties of present methods and extracts information from the data series more efficiently. A fit to the normal distribution is verified, and a computational example is given.

1. INTRODUCTION

After Thom and Shaw (1958) laid the groundwork for a climatological prediction of the first occurrence of low temperatures in the fall and of the last occurrence in spring, Thom (1959) devised an analytic technique to be used when the preselected low temperatures did not occur every year. Thom's methods have been used successfully by a great number of workers, and many "freeze" bulletins have been issued by NOAA climatologists giving the climatological probabilities associated with the first fall and last spring occurrences of temperatures $\leq 32°$, 28°, 24°, 20°, and 16°F in their States. These analyses used Thom's convention of having the spring season running from January 1 through June 30 and the fall season extending from July 1 through December 31.

Unquestionably, this pioneering work has been of great value; but as the years passed, it became apparent to a few that these methods were not completely satisfactory in Florida, the Southeast Atlantic States, and the Gulf Coast States. Here, fairly frequently, one or more years pass without some or all of the low temperature thresholds being reached. This results in a sizable number of zero occurrences during the period of record; and all too frequently, the residual time series is too short to be analyzed meaningfully.

This difficulty arises from the fact that these coastal areas do not have really well-developed fall and spring seasons as do the more Northern States and the Central States. Rather, they experience a cool or cold season that can be very brief or nearly absent in any particular year. In addition, the chances are that most of the cold seasons in these areas will occur after December 31 in any year. According to Thom's seasonal convention, all events of this nature would produce zero occurrences in fall, and only the last such occurrence in spring would be counted; any others between January 1 and the last occurrence would be ignored.

A convenient way around the difficulty is to begin the "cold season" with July 1, end it on June 30, and use time series of the first and last occurrences of each temperature threshold during this period. Besides overcoming the above difficulties, this concept utilizes the information in the data series more efficiently. For instance, it is possible to recognize the fact that, in a certain year, the first and last occurrences of 24°F happened on February 4 or that the first cold season temperature $\leq 24°$F was on January 30 and the last was on February 4. Also, the scheme recognizes that there is no difference of consequence whether a plant freezes on December 31 or January 1 of any particular year. Similar statements can be made concerning the timing of man-controlled activities susceptible to the adverse effects of low temperatures.

2. METHOD OF DATA ANALYSIS

The notation used below follows that of Parzen (1960) and consists of the following:

A is the event a preselected low temperature threshold is reached sometime during the cold season.

A^c is the event a preselected low temperature threshold is not reached during the cold season.

B is the event a preselected temperature is reached before a certain date after the beginning of the cold season.

C is the event a preselected temperature is reached after a certain date before the end of the cold season.

A data series for a particular location can be analyzed to determine the climatological probability of a preselected low temperature before a preselected date; or, conversely, the probability level (the risk the user wants to take) can be selected in advance, and the data series can be used to determine the unknown cold season date associated with this risk. The latter approach appears to be the more useful, and is the one discussed first.

Accordingly, we are interested in fixing $P(B)$ and $P(C)$, which are the probabilities of the events B and C occurring, and determining from the data series the cold season dates associated with preselected probability levels. We chose 0.10, 0.25, 0.50, 0.75, and 0.90 as the five probability levels to be preassigned to $P(B)$ and to $P(C)$; further, we continue the convention adopted by Thom of fixing the low temperature thresholds at 32°, 28°, 24°, 20°, and 16°. Other probability levels or freeze threshold values can be used since the methods developed here are applicable generally.

For any two events, such as A and B, we can write the probability that B will occur as

$$P(B)=P(B|A)P(A)+P(B|A^c)P(A^c) \tag{1}$$

where $P(B|A)$ is the conditional probability that B will occur given that A has occurred. Obviously, $P(B|A^c)=0$ in this particular case. Since $P(B)$ is predetermined,

$$P(B|A)=\frac{P(B)}{P(A)}. \tag{2}$$

Replacing event B with C in eq (1) and (2), we have also

$$P(C|A)=\frac{P(C)}{P(A)}. \tag{3}$$

We estimate $P(A)$ by letting

$$P(A)=m/n$$

where m is the number of cold seasons during which the preselected temperature threshold value was reached or exceeded and n is the total number of years of record used. It is necessary to adopt some convention of assigning day numbers to the dates on which each temperature threshold was reached during the cold season. The convention used assigns day 1 to July 1 and day 365 to June 30. During leap years, the day number assigned to February 29 (244) is the same as the day number assigned to March 1 during nonleap years since the length of the cold season is unaffected by the change in length of the calendar year; subsequent leap-year day numbers are increased by one, also, through June 30.

The NOAA climatologist for Louisiana was generous in providing a listing of the dates on which temperatures of 32° or lower were recorded during a 22- to 32-yr period of record for eight central or southern Louisiana stations (Baton Rouge, Bunkie, Covington, Donaldsonville, Houma, Lafayette, Morgan City, and Schriever). Day numbers were assigned to these dates for events B and C for each of the given temperature thresholds listed earlier. This action provided 80 sets of day numbers, each one of which was tested for its fit to the Gaussian probability distribution using a table (Lilliefors 1967) of critical values for the Kolmogorov-Smirnov goodness-of-fit test. To be 90-percent confident that the normal distribution was acceptable, one could have eight sets of the data failing the test and still accept the fit-to-normal hypothesis. Since only four sets of data failed, the fit-to-normal assumption seems reasonable.

With the acceptance of a Gaussian distribution, it becomes possible to use $P(B|A)$ and $P(C|A)$ from eq (2) and (3) to determine t from a $N(0,1)$ table of the inverse normal distribution (i.e., a table with zero mean and unit variance). Then we can use the expression

$$x = ts + \bar{x}, \tag{4}$$

where

x is the day number associated with $P(B)$ or $P(C)$,
t is the number of $N(0,1)$ standard deviations associated with $P(B|A)$ or $P(C|A)$,
s is the standard deviation of the day numbers of event B or C, and
$\bar{x}$ is the mean day number of event B or C,

to find the date associated with a risk equal to a predetermined $P(B)$ or $P(C)$.

If $P(A)$ is less than a preselected $P(B)$ or $P(C)$, the unknown date x does not exist. For example, if $P(A)=$ 0.87 for the 28° threshold for event C, a date does not exist after which there is a 0.90 probability of occurrence.

If one wishes to fix a date in advance and then determine the $P(B)$ or $P(C)$ associated with this date, the correct precedure is the reverse of that just described. Equation (4) is solved for t; this value is then used to enter a regular table of $N(0,1)$ to determine $P(B|A)$ or $P(C|A)$. Then, eq (2) or (3) is solved for $P(B)$ or $P(C)$ to get the desired probability.

3. COMPUTATION EXAMPLE

The period of record used for the Houma, La., station was the 31-yr period 1938–1939 through 1968–1969. During this period, 13 of the cold seasons were without a temperature of 24° or less. Therefore, $P(A)=18/31=$ 0.581; and the value of $\bar{x}$ of the B events is 184.667 which is the day number of Janaury 1. The following expression was used to calculate the variance:

$$s^2=\frac{\Sigma x^2-\frac{(\Sigma x)^2}{n}}{n-1},$$

the square root of which is

$$s=26.386.$$

Assuming we want the date before which there is only a 25-percent chance of having a temperature as low as 24°, we write

$$P(B\mid A)=\frac{P(B)}{P(A)}=\frac{0.25}{0.581}=0.431.$$

After entering a $N(0,1)$ table of the inverse normal distribution, we find that a value of 0.431 is associated with $t=-0.1738$. These values are then substituted in eq (4):

$$
\begin{aligned}
x &= ts + \bar{x} \\
&= (-0.1738)\,(26.386) + 184.667 \\
&= 180.081
\end{aligned}
$$

which corresponds to December 27. Thus, at Houma, La., there is a 25-percent risk of having a 24° or lower temperature before December 27. Similar calculations reveal there is a 50-percent chance of having a 24° or lower temperature by January 29 at Houma.

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PICTURE OF THE MONTH

Air Pollution Photographed by Satellite

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Contamination of the air we breathe is one of the more serious pollution problems of this decade. The problem of pollution in general is the concern of all sciences; the problem of air pollution is the particular concern of the meteorologist. Ways must be found to measure accurately the amounts and kinds of pollution, to locate their sources, and to devise countermeasures. The meteorological services of many countries have or are in the process of establishing special networks of stations to measure contaminants. Measurements will be made of airborne oxides of sulfur and nitrogen, hydrocarbons, fly ash, and other particulate matter. Sensors now under development for use on satellites may permit daily global assessment of the three-dimensional distribution of these atmospheric contaminants.

Regardless of the future, it is possible to use today's satellite pictures to locate areas of critical pollution potential (i.e., areas with existing low-level inversions). Such areas usually can be detected only under cloud-free conditions in areas where a certain amount of pollution is already occurring. The two ESSA 8 pictures used as figures 1 and 2 show Western and Central Europe under conditions of great contrast. In one, the atmosphere is relatively pollution free; in the other, pollution is prominent. Both were received by APT (automatic picture transmission) directly from ESSA 8 by Deutscher Wetterdienst, Offenbach/Main, Germany.

The first of these pictures (fig. 1) shows a large area of clear skies over North-central and Western Europe on June 18, 1970. The coastlines along the Baltic and North Seas are sharp and distinct; most of England is visible. Central Europe and southern Scandinavia are under the influence of the southeastern edge of an anticyclone centered in the Norwegian Sea. A very dry northeasterly airflow dominated the area; the range of dry-bulb–wet-bulb temperature separations was between 15° and 20°C. Visibilities in northern Germany were reported in the 25- to 40-km range, and maximum temperatures reached 25°C. Note fog forming in the easterly flow over the northern part of the North Sea and the clear area in the lee of northern Scotland and the Shetlands.

The situation on Aug. 5, 1970, was quite different (fig. 2). A ridge of relatively high pressure lying between a Low over the Bay of Biscay and a weak Low over the Baltic Sea (B) dominated Central Europe. The clouds at (A) are part of a frontal system connected with the cyclone over the Bay of Biscay. A sharp inversion based at 900 mb had formed under the weak ridge of high pressure over Central Europe. The coastlines that showed so clearly in figure 1 are obscured in figure 2 by haze in the moist maritime air that was moving onshore. The band (C) extending from the frontal zone at (A) has the appearance of dense cirriform cloudiness. Though only a few of the stations in this area reported cirrus, all recorded dust or haze. Therefore, one may assume that the band (C) represents an extensive area of air pollution with haze and dust trapped below the temperature inversion. In this heavily industrialized region (marked by the band), the effluents emitted by factories and the unfavorable atmospheric conditions (i.e., the existence of a low-level inversion and light surface winds) combined to produce this pollution situation. Maximum temperatures of about 27°C, relative humidity ranging between 70 and 80 percent, and visibilities from 1.5 to 6 km created a very uncomfortable day for the population in this area.

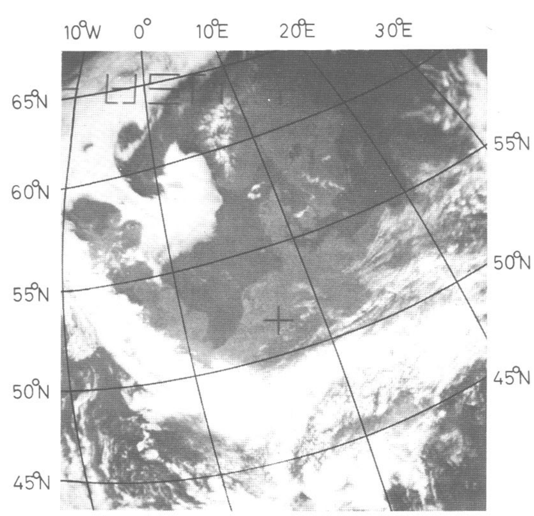

FIGURE 1.—ESSA 8 photograph, pass 6901, at 1001 GMT on June 18, 1970.

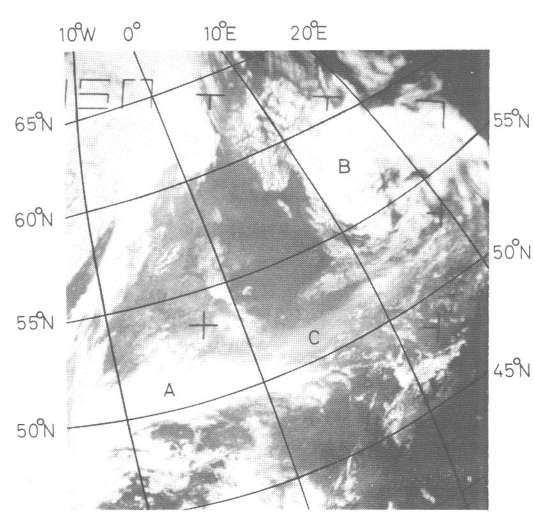

FIGURE 2.—ESSA 8 photograph, pass 7504, at 1046 GMT on Aug. 5, 1970.